

THE CALCULATION OF DIRECT CURRENT ELECTROMAGNETIC CIRCUITS.*

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In the following pages, attention will be paid solely to magnetic circuits in which the flux is either constant, or subject only to gradual or occasional changes in value or direction, or both, so that energy losses due to hysteresis, eddy currents, and other phenomena which are encountered in alternate current electromagnetic circuits may be neglected. Practically all magnetic circuits employed in direct current apparatus may be included in the above classification.

The fundamental law of the magnetic circuit takes the same form as Ohm's law for the electric circuit and is†

$$\phi = \frac{\mathcal{F}}{\mathcal{R}} \quad (I)$$

where ϕ = the magnetic flux in webers (C. G. S. lines),

$\mathcal{F}$ = the magnetomotive force in gilberts,

(one gilbert = .7958 ampere turn) and

$\mathcal{R}$ = the magnetic reluctance in oersteds.

(one oersted = the reluctance of a centimetre cube of air pump vacuum between opposed faces.)

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†H. A. Rowland, Electrician, vol. 13, p. 536.

Depending upon work of Gauss, Ampere, Maxwell*, and others, we know that

$$\mathcal{I} = \frac{4\pi Ni}{10}, \quad (2)$$

where N is the number of convolutions of conductor in the coil carrying the magnetizing current, i , expressed in Amperes. We also have the value of magnetomotive force expressed in gilberts by

$$\mathcal{I} = \mathcal{H} l, \quad (3)$$

where $\mathcal{H}$ is the magnetizing force per unit length in vacuum, acting over the length l . This leads to an expression for the number of ampere turns in the magnetizing coil,

$$Ni = \frac{\mathcal{H} l}{1.257} \quad (4)$$

Or, if the length of magnetic circuit be expressed in inches (l_1) in place of centimetres,

$$Ni = \frac{\mathcal{H} l_1}{.495}. \quad (5)$$

The reluctance of a magnetic circuit is analogous to the resistance of an electric circuit, so that for a complex series magnetic circuit of different cross sections and built of materials having different specific reluctances, the equivalent reluctance of the entire circuit may be expressed by

$$\mathcal{R} = \Sigma \frac{1}{A\mu}, \quad (6)$$

where the summation of the reluctances, $\frac{1}{A\mu}$, for *each* portion of the circuit is taken with dimensions of area, A , and length, l , in centimetres, and μ , the permeability, is the recip-

*Mascart & Joubert, *Electricity and Magnetism*, vol. 1, §§ 62, 450.

J. C. Maxwell, *Treatise on Electricity and Magnetism*, vol. 2, §§ 407, 409.

rocal of the specific reluctance of the material composing the particular portion of the circuit considered.

We may now write the more general expression for the magnetic circuit,

$$\phi = \frac{\frac{4\pi Ni}{10}}{\sum \frac{1}{A\mu}}, \quad (7)$$

which gives, as the necessary number of ampere turns for the magnetization of any series circuit,

$$Ni = .7958 \phi \cdot \sum \frac{1}{A\mu}, \quad (8)$$

where all the dimensions are in centimetres. If all the dimensions are taken in inches this becomes*

$$Ni = .3132 \phi \cdot \sum \frac{1}{A\mu}. \quad (9)$$

All materials of construction, with the exception of iron, cobalt, nickel and their alloys, have practically the same specific reluctance, as air or vacuum at the ordinary temperatures†. What are known as the magnetic metals have much lower reluctances, so that while we may consider, for practical purposes, the permeability of all other substances as unity, the values of permeability for iron, and particularly the soft grades of wrought iron and steel, are much higher.

For any material the flux density, or magnetic induction, in the magnetic circuit will be

$$\mathcal{B} = \mu \mathcal{H}, \quad (10)$$

so that, except in magnetic substances, the number of gausses, B , (C. G. S. lines per square centimetre) is

*S. P. Thompson, The Electromagnet, p. 117.

†J. A. Ewing, Magnetic Induction in Iron and Other Metals. Chap. 8.

numerically equal to $\mathcal{H}$, the field intensity or magnetizing force.

The values of $\mathcal{B}$ and $\mathcal{H}$ employed in the ferric magnetic circuits of commercial apparatus seldom exceeds 16,000 and 50 respectively, and their relative values are best shown by magnetization curves determined experimentally.* The accompanying plate gives a number of curves which are representative of commercial irons, and may be used as a basis of calculation for magnetic circuits, where tests on samples of iron to be used can not be obtained.

MAGNETIZATION—PERMEABILITY CURVES.

No. of Curve.	Material.	Determined by
1.	Annealed Wt. Iron,	John Hopkinson.
2.	Mitis Casting(average),	M.E.Thompson.†
3.	Cast Steel,	H. B. Smith.
4.	Sheet Steel,	H. B. Smith.
5.	Sheet Iron,	M.E.Thompson.†
6.	Cast Steel (average),	M.E.Thompson.†
7.	Malleable Iron,	L. W. Downes.
8.	Special Cast Iron,	M.E.Thompson.†
9.	Soft Cast Iron(average)	H. B. Smith.
10.	Grey Cast Iron,	John Hopkinson.
11.	Ordinary Foundry Cast Iron (average)	H. B. Smith.
12.	Permeability curve ($\mathcal{B}$ and μ) for No. 3.	
13.	“ “ “ “ “ “ “	4.
14.	“ “ “ “ “ “ “	5.
15.	“ “ “ “ “ “ “	9.

*J. A. Ewing, Magnetic Induction in Iron and Other Metals. Chap. 3.
Ewing and Klaassen, Electrician, vol. 32, p. 636.

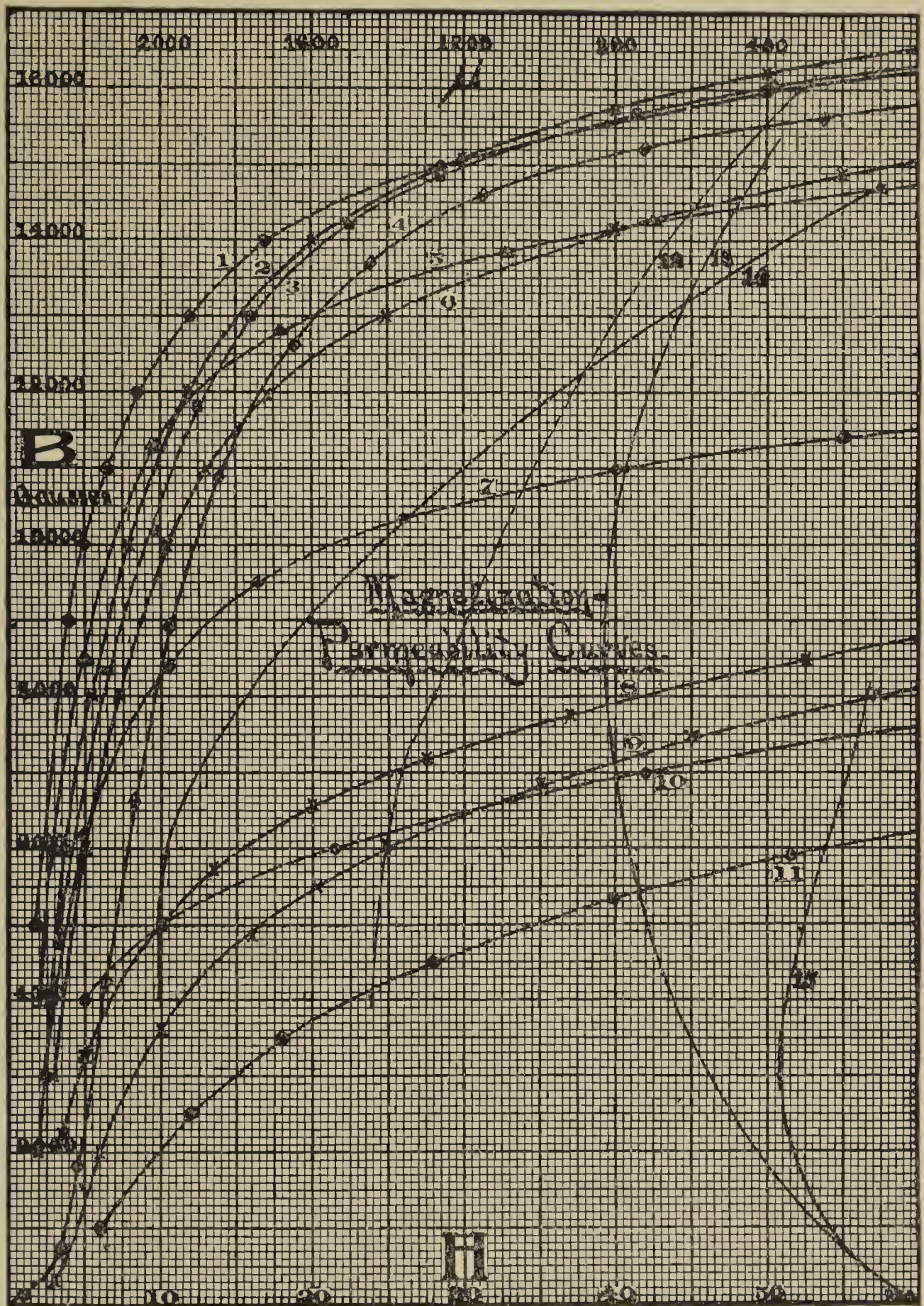
J. Fischer-Hinnen, Elektrischer Gleichstrom-Maschinen, p. 160.

H. du Bois, The Magnetic Circuit. Chap. 11.

S. P. Thompson, The Electromagnet. Chap. 3.

Treat and Esterline, Electrical World, vol. 30, p. 696.

†M. E. Thompson, P. H. Knight and G. W. Bacon, The Magnetic Permeability of Special Irons for Electrical Purposes. Trans. A. I. E. E., vol. 9, p. 250.



For any cross section of a magnetic circuit,

$$\phi = \mathcal{B} A, \quad (11)$$

and in all pieces of apparatus these values are definitely fixed, at some one section at least, depending upon the service demanded. For example, in a dynamo or motor, ϕ is fixed by the fundamental equation for electromotive force and $\mathcal{B}$ or $\mathcal{H}$ at the air gap can not go outside quite definite limits for a particular machine. Then for any other sections of the same magnetic circuit, neglecting magnetic leakage, the cross section will vary inversely with the values of $\mathcal{B}$ that can be economically employed in the material selected at the section of the circuit considered. In general, it may be said that the best and most economical results are secured in magnetic circuits including an air gap where the values of $\mathcal{B}$ are taken just below the knee of the curve of magnetization*, but this is not always the case, as will be found in magnetic circuits for portative magnets, compound dynamos and constant speed shunt motors. For traction magnets and most electromagnetic mechanisms, the above condition should usually be observed.†

In the magnetic circuits of dynamos, motors, electromagnets and solenoids of various types, the value of the induction at the air gap is usually fixed by the conditions of operation and forms the starting point for calculations. For traction magnets, this value is determined from the law of magnetic traction stated by Maxwell§ as

$$P = \frac{\mathcal{H}^2 A}{8 \pi}, \quad (12)$$

*W. E. Goldsborough, Economy in the Design of Electromagnets. Electrical World, vol. 29, p. 196.

†E. R. Carichoff, Design of Electromagnets for Specific Duty. Electrical World, vol. 23, pp. 113, 212.

§J. C. Maxwell, Treatise on Electricity and Magnets, vol. 2. § 643.

where P is the traction in dynes and A is the area of the polar surface in square centimetres. The experimental verification of this law by Bosanquet* and others† justifies its direct practical application in the more convenient forms,

$$P_1 = \frac{\mathcal{B}^2 A}{24,655,180}, \quad (13)$$

where P_1 is the traction in kilogrammes, and

$$P_2 = \frac{\mathcal{B}^2 A}{11,183,460}, \quad (14)$$

where P_2 is the traction in pounds; the area in both cases being in square centimetres. As the tension along the lines of force becomes evident at the air gap, where the permeability is unity, $\mathcal{B}$ is numerically equal to $\mathcal{H}$ and may be used as above. Depending upon these expressions, the accompanying plate has been calculated and plotted for use in the rapid practical calculation of magnetic traction in ordinary magnetic circuits. Results obtained from the use of the plate are sufficiently accurate for all practical purposes.

The above expressions assume uniform distribution of flux over the area of section considered and such assumption is sufficiently accurate in most practical circuits. Where the flux distribution is not uniform, the more general expression of equation (12) becomes

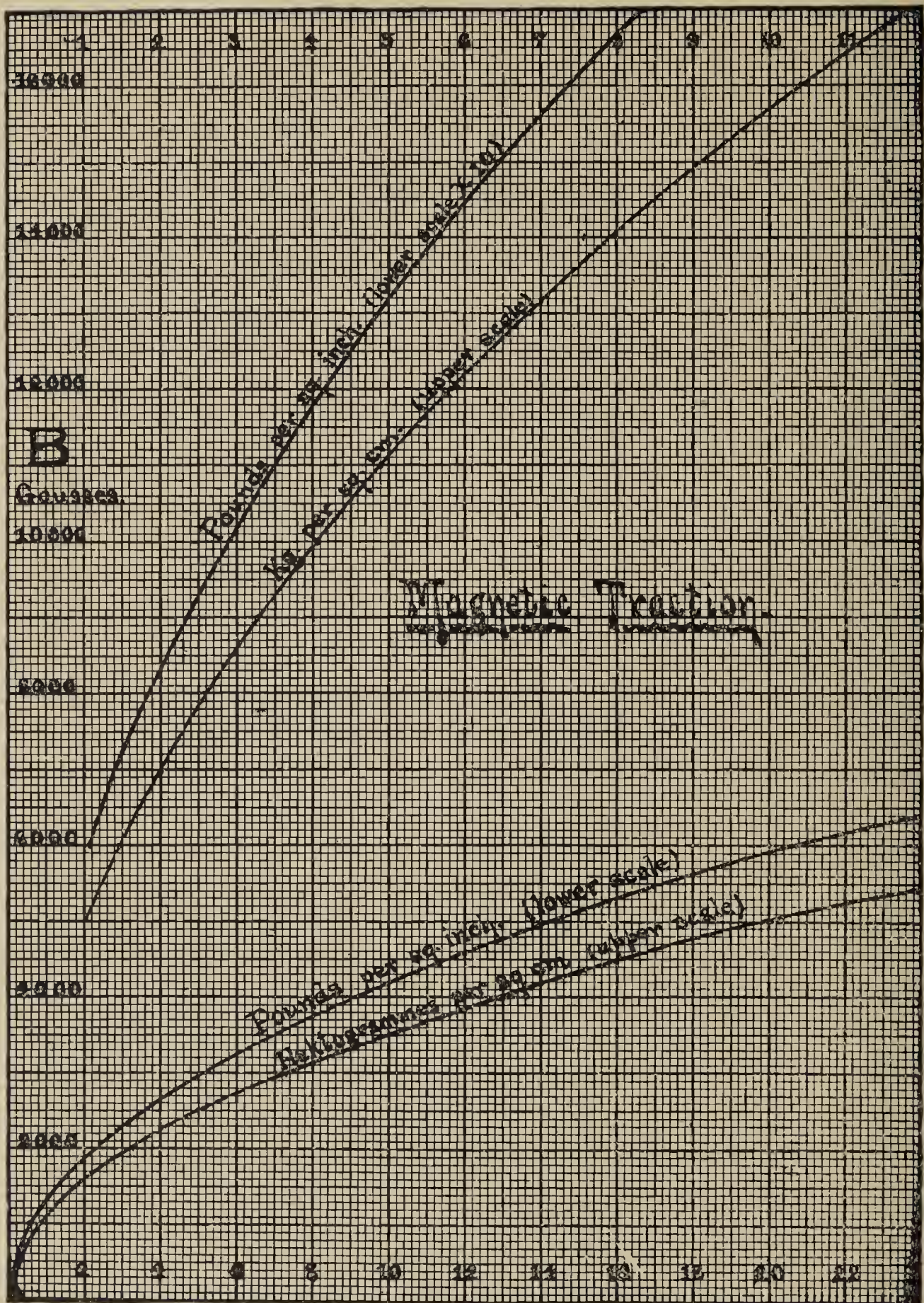
$$P = \frac{1}{8\pi} \int \mathcal{H}^2 dA, \quad (15)$$

and this expression, and those that may be developed from it, can be used in the occasional cases where the preceding expressions are not applicable.

*Phil. Mag., vol. 22, p. 535. Electrician, vol. 18, Dec.

†R. Threlfall, Phil. Mag., vol. 38, p. 89.

E. T. Jones, Phil. Mag., vol. 39, p. 254. Ibid., vol. 38, p. 89.



From the preceding, it is seen that, for a given total flux, the smaller the area of cross section of magnetic circuit across which attraction takes place, and consequently the higher the induction employed, the greater will be the magnetic attraction produced between the surfaces.* This is a question of increasing the attraction with a given flux and must not be confused with the condition of maximum economy previously mentioned.

We are now able to calculate the number of ampere turns necessary for the excitation of a simple electromagnet, of uniform cross section and permeability, to produce a given attraction between its poles and armature. Assuming the joints in the circuit so perfect as not to increase the reluctance and that there is no magnetic leakage, we have, from equation (8), for unit area of cross section

$$\mathcal{B} = \frac{\mu N_i}{.7958 l}, \quad (16)$$

and from equation (13),

$$\mathcal{B} = 4967 \sqrt{\frac{P_1}{A}}. \quad (17)$$

Equating these expressions,

$$N_i = \frac{3952 l}{\mu} \sqrt{\frac{P_1}{A}}, \quad (18)$$

where all dimensions are in centimetres and the pull is expressed in kilogrammes, per square centimetre, or,

$$N_i = \frac{2661 l_1}{\mu} \sqrt{\frac{P_2}{A_1}}, \quad (19)$$

where all dimensions are in inches and the pull is in pounds per square inch.

*S. P. Thompson, *The Electromagnet*, p. 131.

H. DuBois, *The Magnetic Circuit*, p. 168.

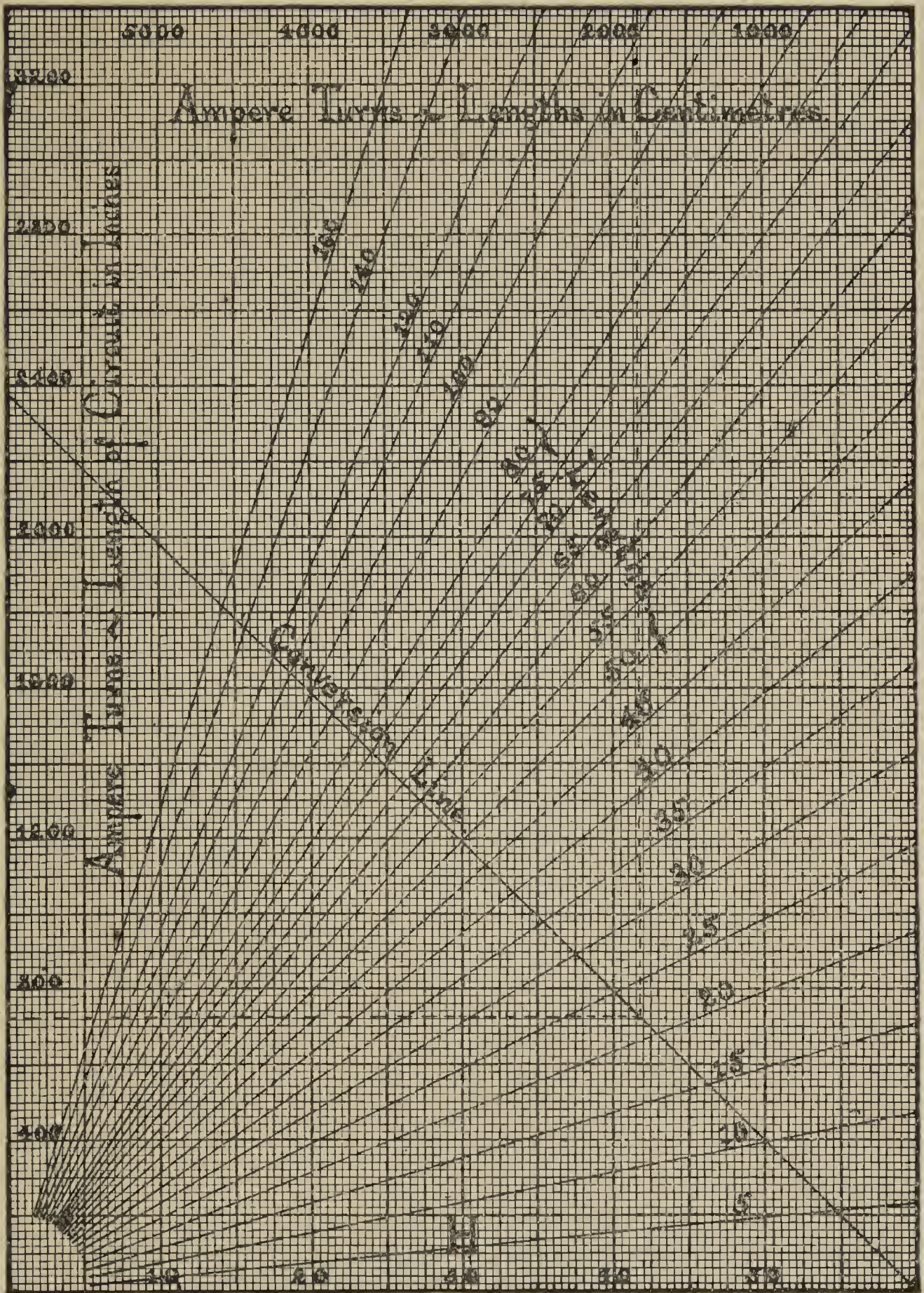
Ind. and Iron, Jan. 12, 1894.

Practical magnetic circuits will not ordinarily be subject to conditions which make these expressions directly applicable, because of imperfect joints, magnetic leakage, and varying cross section and magnetic material from which the several parts of the circuit are constructed. The following expression is, however, used directly for practical calculation and involves a knowledge of the lengths of the several portions of the circuit and the values of $\mathcal{N}$ corresponding to the induction in each of those parts, after having made allowance for magnetic leakage. From equation (4), the total ampere turns necessary for a complex circuit are

$$Ni = \sum \frac{\mathcal{N}_1 l_1}{1.257} + \frac{\mathcal{N}_2 l_2}{1.257} + \dots + \frac{\mathcal{N}_n l_n}{1.257}. \quad (20)$$

To avoid the considerable amount of computation involved in the calculation of common magnetic circuits by this expression, reference may be made to the accompanying plate which will give results sufficiently accurate for most calculations. To illustrate its use, a few cases will be considered.

A section of a magnetic circuit has a length of 30 centimetres and is made of cast steel of such dimensions as to give an induction of 15000 gaussses. From curve 3 of the first plate this gives a value of $\mathcal{N}$ of 30.5. Find the location of this value on the computation plate and the intersection of a vertical through this point with the diagonal marked 30 gives a point which, if we pass horizontally to the left, is found to correspond to 722 ampere turns. (Arithmetical calculation gives 728.) Suppose this section is in series with an air gap 5 millimetres in length in which the induction is 5000 gaussses, whence $\mathcal{N}$ is 5000. The results given by the plate may be used when the horizontal scale for $\mathcal{N}$ is multiplied or divided by a number, if the values of the diagonal lines are divided or multiplied by the same



number. In this case, multiply the horizontal scale by 100 and divide the diagonal line values by 100. The intersection, then, of the vertical through 50 with the diagonal 50 gives, by reference to the scale at the left, 1985 ampere turns necessary. (Arithmetical calculation gives 1988.) The total ampere turns, then, for the two portions of the circuit as obtained from the plate are 2707, which is within .3 per cent. of the correct value. In case the length of the magnetic circuit is given in inches, the only difference in the use of the plate is that, having found the intersection with one of the numbered diagonal lines, as above, you then pass horizontally to the intersection with the conversion line and then vertically to the scale at the top of the plate, the conversion line serving to multiply the values on the scale at the left by 2.54, the conversion factor between the inch and centimetre. With $\mathcal{N}$ equal to 30.5 and the length of the circuit 30 *inches*, the use of the plate as indicated gives 1825 ampere turns necessary.

The graphical method of calculation outlined can be depended upon for results within one per cent. or less. Where much work of this sort is undertaken, a large plate giving results within one-tenth of one per cent. is of great convenience, and is quickly constructed.

With the exception of the effects of leakage and joints in the magnetic circuit, which will be considered later, the above covers the fundamental basis of calculation of such circuits, but there are many points of decided practical importance which have not been developed.

The proportioning of the circuit to carry the required flux at economical densities for the materials selected for the several parts of the circuit has been considered in connection with equation (11) and must depend upon the experience of the designer or on comparative sets of calcula-

tions. In general, it may be said that the mean length of the circuit should be made as short as possible and yet enclose the windings or other essential features of the design of which the magnetic circuit forms a part. The proportioning of the various parts and surfaces of the metal forming the magnetic circuit and adjacent parts of the mechanism should be carried out so as to reduce the magnetic leakage to a minimum compatible with other considerations. The fact that the development of the total flux usually requires so small a percentage of the total energy of a mechanism or system, indicates that to secure a slight reduction in the leakage it is not generally advisable, on this account, to sacrifice other important features to any considerable extent.

The selection of the proper material for the parts of the circuit involves the questions of first cost of material, costs of manufacture and the efficiency of the completed mechanism. In most instances, it may be said that cast steel or sheet iron or steel make the most desirable materials in all respects. Not only is cast steel of so much greater permeability than cast iron as to make it a cheaper material to use with ordinary shop and market conditions, but the reduced area of section occasioned by its use reduces the weight of copper and the expenditure of energy necessary in the existing coils, thus further reducing the cost and improving the efficiency of the completed mechanism.* In many cases, the cost of materials and shop processes are such as to make laminated fields, built up of sheet steel, even cheaper than if built of cast steel, and they are always cheaper than small iron castings, if a sufficient number of fields are built to reduce the cost of punch and die per field to a low value.

*A. D. Adams, The Best Metal for Field Magnet Frames. Trans. A. I. E. E., vol. 12, p. 18.

The correct mechanical design of the purely mechanical features of the magnetic circuit is important. Its section must be able to endure the stresses likely to occur in operation, and, at the same time, present a correct and finished appearance that is pleasing to the eye. Since the introduction of cast steel for magnetic circuits, it has been found in many instances that the section necessary for the magnetic flux is not sufficient to provide mechanical strength, and, on account of this and other reasons based on economical construction, ribbed sections for magnetic circuits are frequently employed. The consideration of the correct mechanical design has frequently been lost sight of by the designers of electrical apparatus in the past.

The effect of the introduction of joints in a magnetic circuit has been shown by Ewing* to be about as follows. Where the surfaces at the joint are scraped true, the reluctance caused by the presence of the joint in the circuit is equivalent to an air gap of from .0025 cm. (.001 inch) at B equal to 4000 gauss, to about .0035 cm. (.0014 inch) at 12000 gauss. Where the surface is not scraped, but is such as is produced by the finish cut of a machine tool, the reluctance of the joint may be taken as an air gap of from .0036 cm. (.0014 inch) at 8000 gauss to .0052 cm. (.002 inch) at 15000 gauss. The reluctance of a scraped joint under compression is almost zero and the above reluctances are frequently negligible in comparison with others present in the circuit. In the case of dynamos, motors, and traction magnets acting over a considerable distance, the reluctance of the joint may be conveniently included in that of the principal air gap.

*J. A. Ewing, *Magnetic Induction in Iron*, pp. 276, 281.

The calculation of the magnetic leakage of electromagnetic circuits is a subject that has been treated by many writers,* but the following method is based principally upon the work of Mr. Wiener† as the easiest of application, and yet sufficiently accurate for practically all engineering calculations.

In magnetic, as in electric circuits, when dealing with series grouping of the circuits, it is most convenient to handle the reluctances and resistances by simple addition of their values for the several parts of the circuit, but when dealing with parallel circuits, it is better to consider the reciprocals of these values, permeances and conductances, as we can, in this way, take the total permeance as the sum of the values for each of the parallel branches. From equation (6), the permeance of a circuit is

$$p = \frac{\mu A}{l}, \quad (21)$$

and, as the permeability of air is unity, the permeance of any path in air becomes the ratio of its mean area of sections normal to the flux and the mean length of the path expressed in the same system of measurement.

There are three primary forms of leakage path, and the leakage of nearly all magnetic circuits can be calculated from the permeances of such paths arranged so as to be the equivalent of the total leakage path of the circuit.

The permeance between two plane rectangular areas, A and A_1 , inclined at an angle of θ degrees with each other, with d the distance between their nearest edges, and l and

*Gisbert Kapp, *Electric Transmission of Energy*, p. 122.

George Forbes, *Tom. Soc. Teleg. Engineers*, vol. 15, p. 551.

S. P. Thompson, *Dynamo-Elect. Machinery*, vol. 1, p. 155.

J. A. Kingdon, *Applied Magnetism*, Chap. 9.

†A. E. Wiener, *Electrical Engineer*, vol. 18, p. 141.

l_1 the distances from those edges to the centres of the respective areas, is

$$p = \frac{.5(A + A_1)}{d + \frac{\pi(l + l_1)}{2} \times \frac{\Theta}{180^\circ}} \quad (22)$$

The permeance between two parallel cylinders of length l and diameter d , with a distance between centres of $d + d_1$, is

$$p = \frac{\pi d l}{d_1 + .785d} \quad (23)$$

The permeance between two parallel half-cylinders of length l and diameter d , with a distance between centres of $d + d_1$, is

$$p = \frac{\pi d l}{2(d_1 + .29d)} \quad (24)$$

From these expressions, and occasional modification to meet special conditions, it is possible to calculate the permeance by any desired path. The leakage factor λ is then obtained as

$$\lambda = \frac{\Sigma p_2}{\Sigma p_1}, \quad (25)$$

where Σp_2 is the permeance by *all* paths and Σp_1 is the permeance by the *useful* paths.

In all the above, the actual difference of magnetic potential by any path as compared with the other paths should be considered. If a series magnetic circuit is excited by two coils, each of which creates an equal difference of magnetic potential in the circuit, then the potential creating leakage about *one* of them is but one-half, and the actual leakage but one-half as much as would be indicated by the direct application of the expressions given for permeance. That is, the leakage about one of the coils is one-half as great as would occur by a path of equal permeance about

the two coils giving double the magnetic potential at the terminals of the path. When this condition exists, perhaps the most convenient way of including the effect of this difference of magnetic potential in the circuit in the calculation of λ is to place a factor (usually 2) in the denominator of the expressions for permeance by the path where the effect occurs. Between any two points in a magnetic circuit, short circuited by a mass of metal of small reluctance, there is small, and usually negligible, difference of magnetic potential.



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